



Lecture 16

Homework



Homework #5

- Yariv and Yeh
 - Problems 10.7, 10.11, 11.2, 11.8
- A p-i-n photodiode is able to convert a pulse of light with 8×10^{12} photons into 3×10^{12} electrons that contribute to the output photocurrent. Calculate the quantum efficiency η and the responsivity R at $\lambda_0 = 0.83 \mu\text{m}$, $1.3 \mu\text{m}$ and $1.55 \mu\text{m}$. Now assume that the photodiode is composed of $\text{In}_{0.70}\text{Ga}_{0.30}\text{As}_{0.64}\text{P}_{0.36}$ and that the intrinsic region perpendicular to the incident photons is $1 \mu\text{m}$ thick. Use Figure 11.15 for absorption as a function of wavelength for different material systems/compositions to estimate the quantum efficiency and responsivity at $\lambda_0 = 1.3 \mu\text{m}$.
- A silicon p-i-n photodiode operating with 0dBm input at $0.8 \mu\text{m}$ has 20MHz bandwidth, 65% quantum efficiency, 1nA dark current and 8pf junction capacitance.
 - (a) Determine the RMS current noise due to shot noise.
 - (b) Determine the SNR due to shot noise.
 - (c) If we require an SNR of 20dB, calculate the minimum received optical power when shot noise is the only noise source.

Detecting Photons (7)

- If we assume a constant rate parameter over the time interval T (independent of j), then the photo-generated current can be written as

$$i(t) = \lambda(t)q$$

$$\lambda(t) = \frac{N}{T}$$

- Then the photocurrent produced by the photodetector can be written in Amperes, assuming the observation time is normalized to one second

$$\begin{aligned} i(t) &= \lambda(t)q = \frac{\eta q}{h\nu} P_{rcvd}(t) \\ &= \Re P_{rcvd}(t) \end{aligned}$$

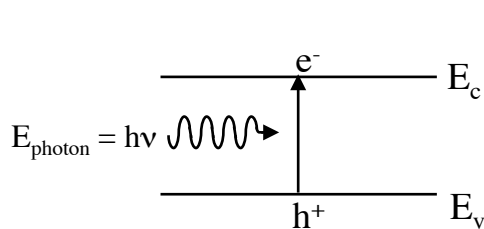
- Where we have defined the detector responsivity as

$$\Re = \frac{\eta q}{h\nu}$$

Photoconductors

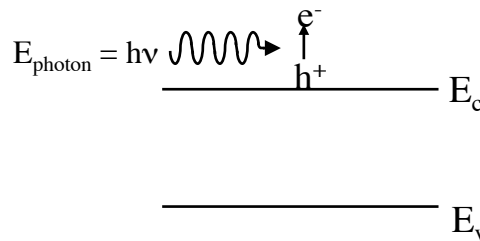
- ⇒ Photon absorption in semiconductor materials.
- ⇒ Three main absorption mechanisms: Intrinsic (band-to-band), Free-Carrier Absorption and Band-and-Impurity Absorption
- ⇒ Intrinsic (band-to-band) is the dominant effect in most SC photoconductors

Intrinsic (band-to-band)

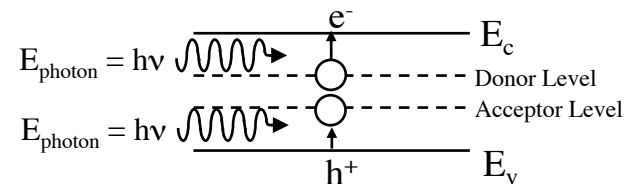


• Incident photon $E_{\text{photon}} = h\nu = E_c - E_v$

Free-Carrier Absorption



Band-and-Impurity Absorption



Photoconductors



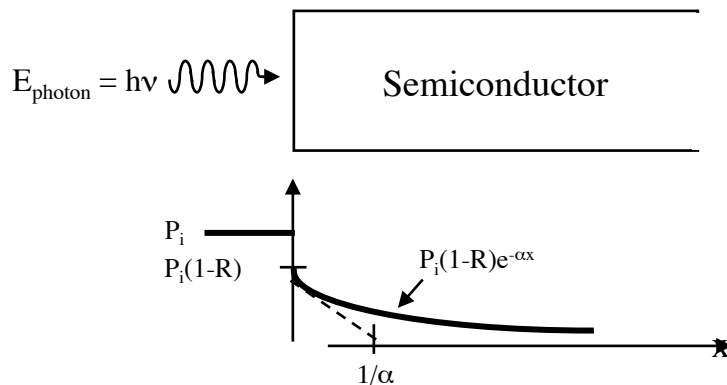
⇒ For intrinsic absorption, photons can be absorbed if

$$\lambda(\mu m) > \frac{hc}{E_c - E_v} = \frac{1.24}{E_g(eV)}$$

$$\lambda(nm) > \frac{1240}{E_g(eV)}$$

Material	Bandgap (eV)	Maximum λ (nm)	Typical Operating Range (nm)
Si	1.12	1110	500-900
Ge	0.67	1850	900-1300
GaAs	1.43	870	750-850
$\text{In}_x\text{Ga}_{1-x}\text{As}_y\text{P}_{1-y}$	0.38-2.25	550-3260	1000-1600

Photoconductors



⇒ Define:

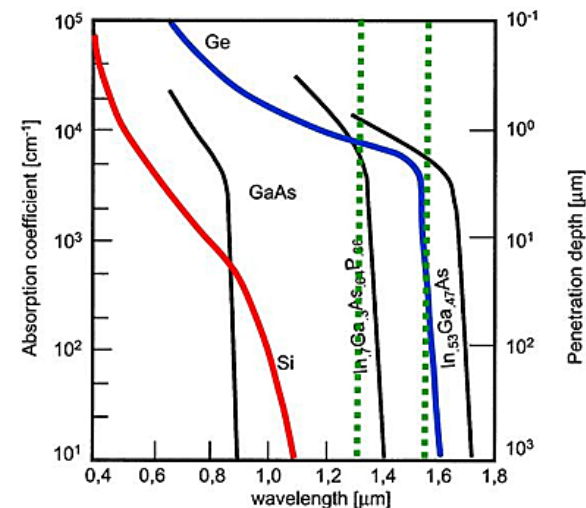
- ⇒ P_i = incident optical power
- ⇒ $R(\lambda)$ power reflectivity from input medium to semiconductor
- ⇒ $\alpha(\lambda) = 1/e$ absorption length
- ⇒ $1/\alpha(\lambda)$ = penetration depth

⇒ Power absorbed by the semiconductor is

$$\begin{aligned} P_{\text{abs}}(x) &= P_i(1 - R)(1 - e^{-\alpha(\lambda)x}) \\ &= \eta(\lambda, x)P_i \end{aligned}$$

⇒ defining the efficiency

$$\begin{aligned} \eta(\lambda, x) &= \frac{\text{number of photocarriers produced}}{\text{number of incident photons}} \\ &= (1 - R)(1 - e^{-\alpha(\lambda)x}) \\ 0 &\leq \eta(\lambda, x) \leq 1 \end{aligned}$$



Photoconductive Photodetectors

- Photogenerated current will have time and wavelength dependence

$$i_{photo}(t) = \frac{\eta q}{h\nu} GP_{rcvd}(t) + i_{dark}$$

$\tau_{carrier}$ = mean free carrier lifetime

$\tau_{transit}$ = transit time between electrical contacts

$$G = \left(\frac{\tau_{carrier}}{\tau_{transit}} \right) = \text{photoconductive gain}$$

i_{dark} = dark current

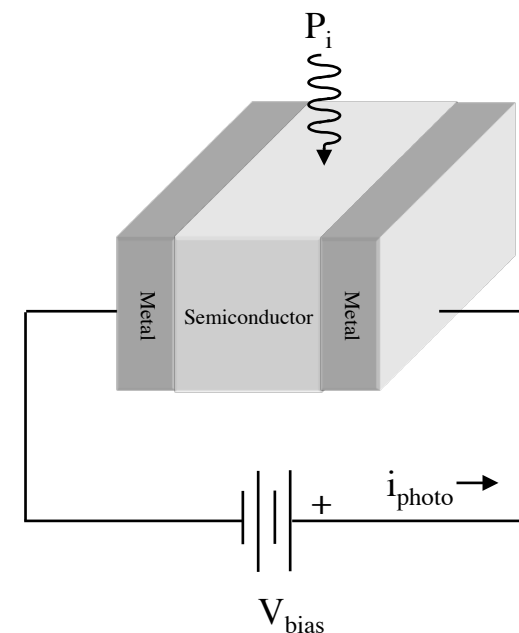
- The transit time for electrons and holes can be different and in many SCs the electron mobility is greater than that of the hole

$$v_e = \mu_e E > \mu_h E = v_h$$

- The SC must remain charge neutral, for every electron generated, multiple holes will get pulled in until the photogenerated electron reaches the other contact. The carrier and transit times are limited by the slower carrier and the photoconductive gain is given by the ratio of the transit times

$$\tau_{carrier} = \frac{L_a}{v_h}$$

$$\tau_{transit} = \frac{L_a}{v_e}$$



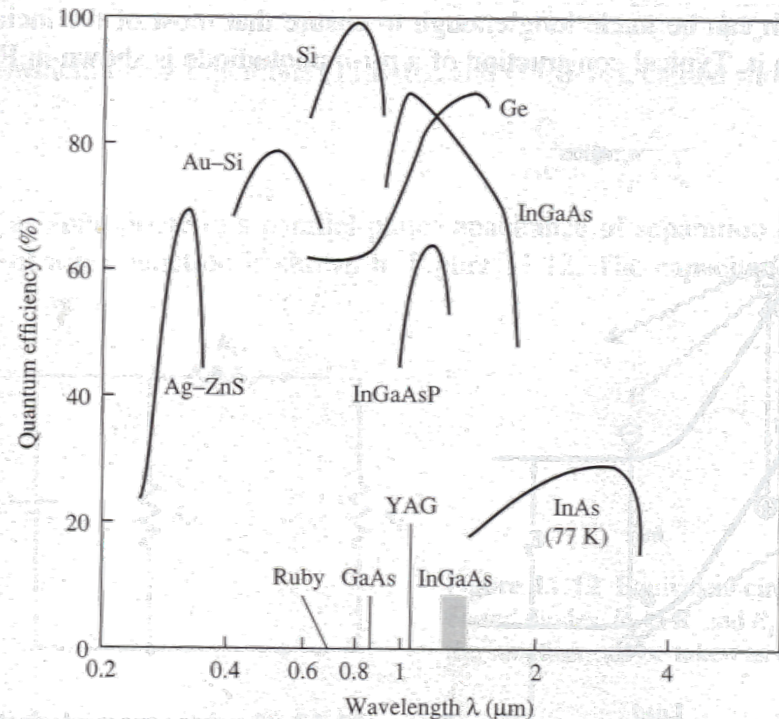
Photoconductive Photodetectors

- The carrier velocity is a linear function of electric field strength up to a saturation velocity (which is the same for both electrons and holes)
 - Field strength of about 10^5 V/cm result in velocities in range of 6×10^6 to 10^7 cm/s
 - Some materials have an electron drift velocity that peaks at 2×10^7 cm/s at 10^4 V/cm
- When photoconductive gain is desirable, detector is operated at low voltages
- Carrier lifetime also impacts the frequency response of the photoconductive photodetector

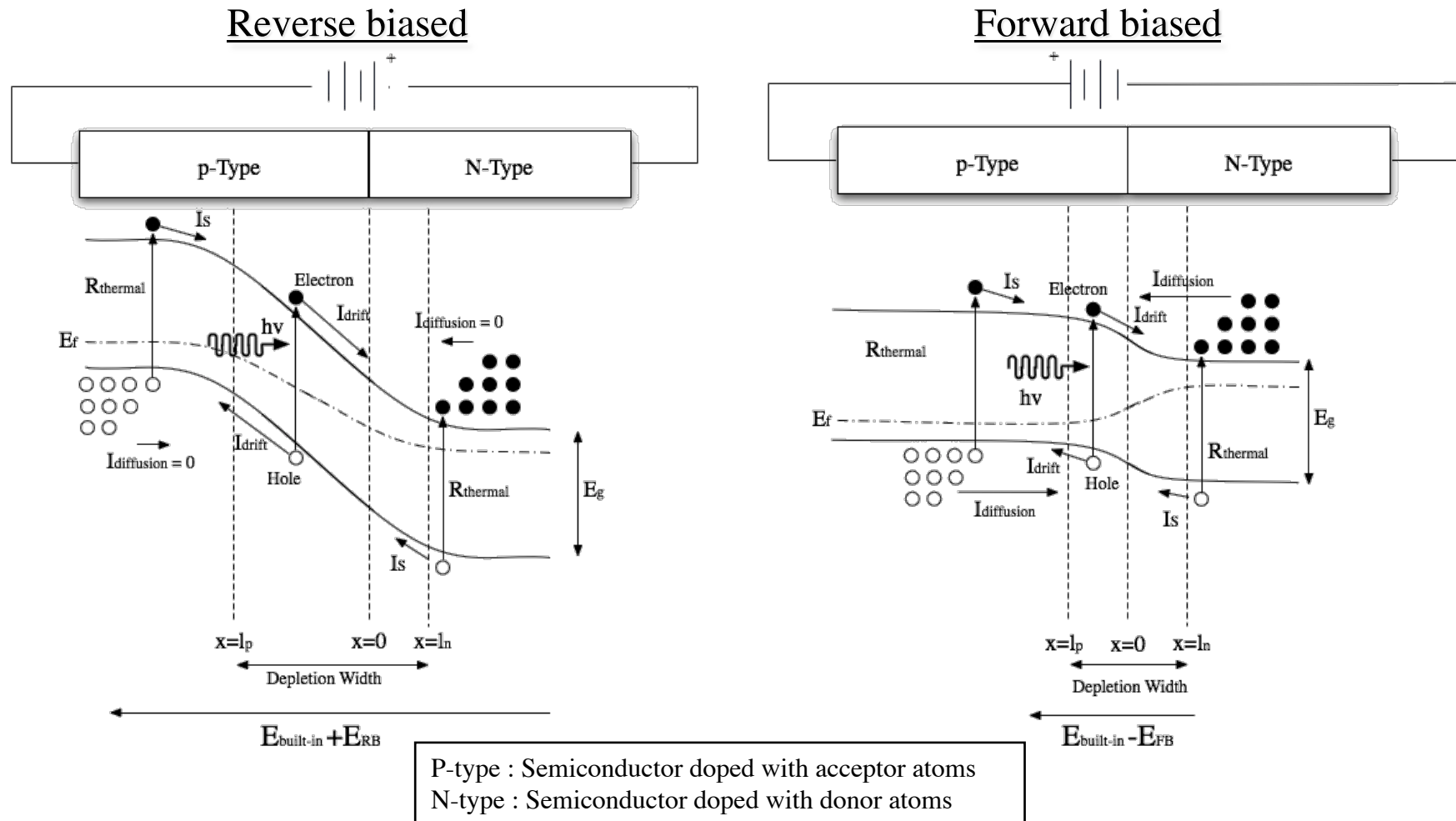
$$i_{photo}(\omega) = \Re G \frac{P_{rcvd}(\omega)}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$$
$$\omega_c = \frac{1}{\tau_{carrier}} = \text{cutoff frequency}$$

Pn Junction Photodetectors

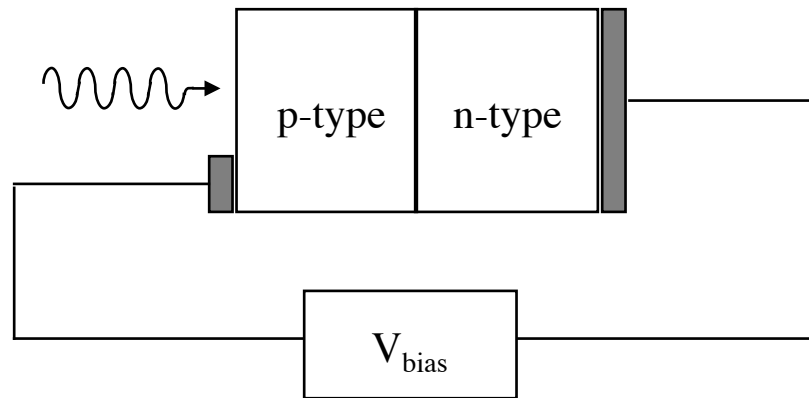
- Photons absorbed within the junction diffusion length generate carriers that are swept down the barrier (drift current) adding to the thermal generation current (dark current) that is present in the absence of light.
- The semiconductor bandgap energy sets a lower limit on the frequency of light that can be absorbed, hence generating photocarriers.
 - Light with energy $h\nu < E_g$ will not be absorbed and will not contribute to photocurrent
 - Light with energy $h\nu \gg E_g$ will be absorbed away from the
- The quantum efficiency and therefore the responsivity are wavelength dependent



Biased p-n Junction Photodiodes



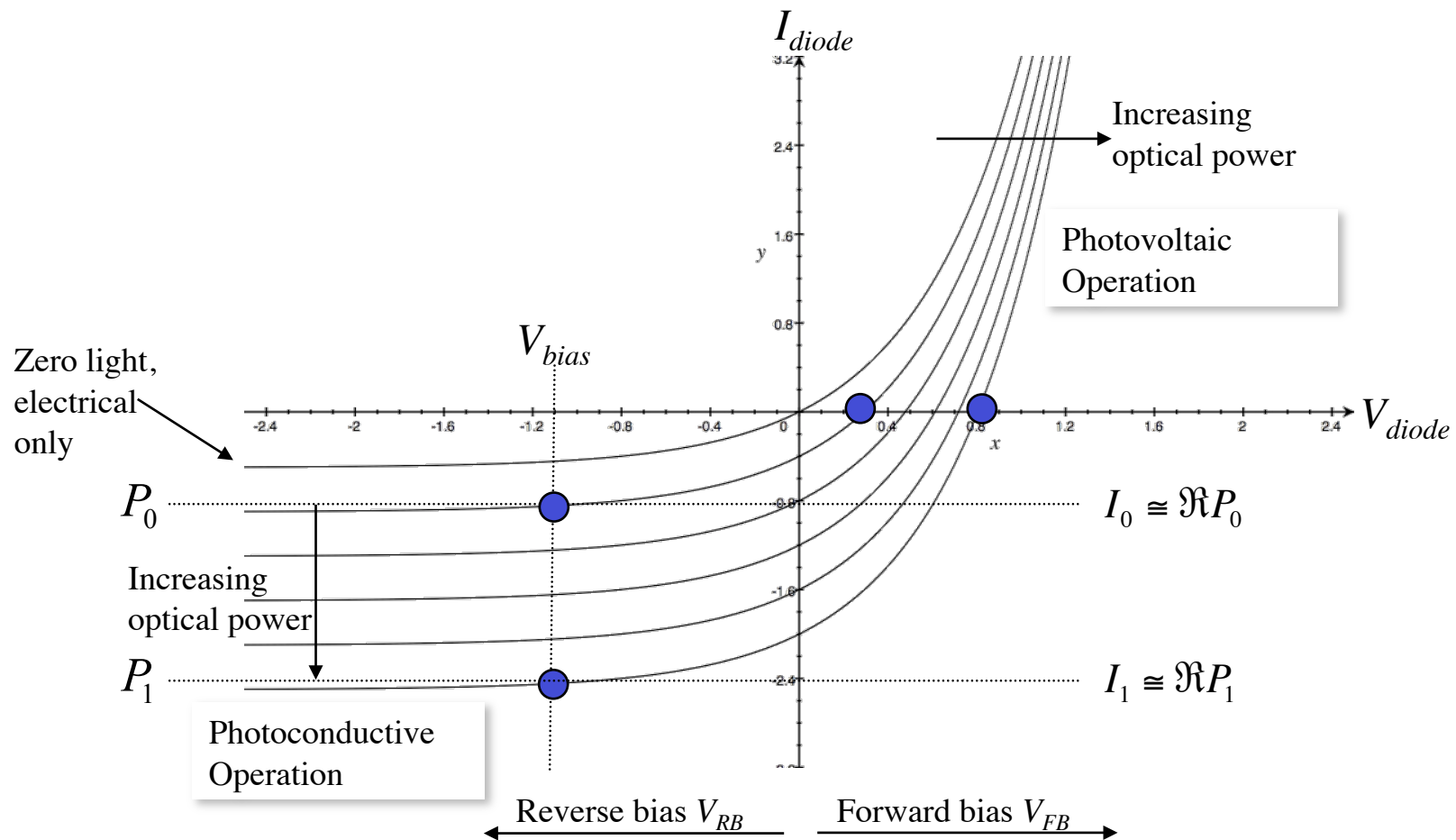
p-n Junction Photodiode Equation



$$I = (I_s) [\exp^{qV_{bias}/K_B T} - 1] - I_{photo}$$
$$= I_{dark} - I_{photo}$$

- I_{dark} = is the current that occurs with zero optical input
- $I_s = I_{th}$ is the thermal or saturation current that occurs in normal (non-illuminated) diode operating mode
- I_{photo} is photo-generated current = $\frac{\eta q}{h\nu} P_{rcvd}$
- q is the electron charge
- V_{bias} is applied bias voltage (positive = forward, negative=reverse)
- K_B is Boltzman's constant
- T is temperature (usually in Kelvin, depending on units of K_B)

p-n Junction Photodiode Regions of Operation



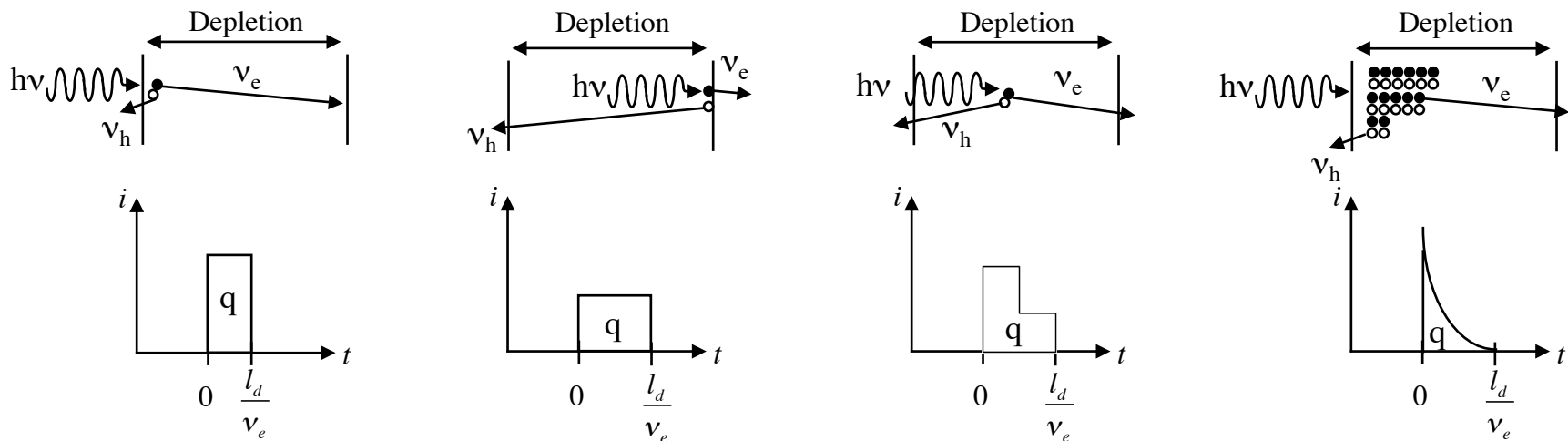
Photodiode Frequency Response



- The ability for the photocurrent to keep up with various modulation frequencies of the optical signal is an important metric in signal and data recovery
- There are three main effects that limit the detector frequency response
 - Finite diffusion time of photocarriers produced in the p and n doped regions of a pn-junction photodiode.
 - RC time constant associated with the pn-junction capacitance
 - Finite transit time of photocarriers drifting across the depletion layer

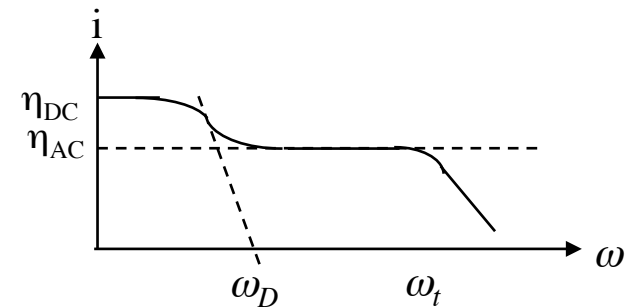
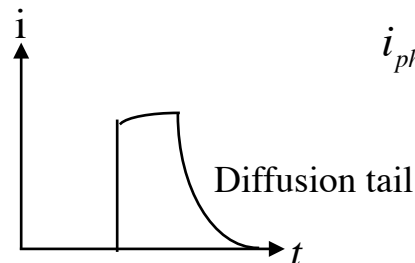
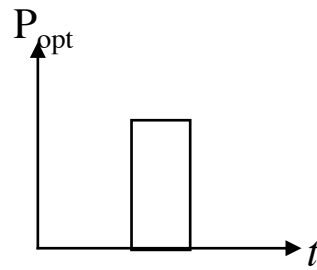
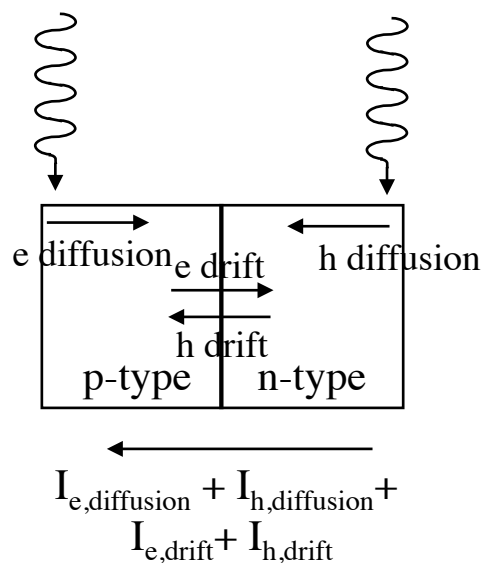
pn-Junction Carrier Dynamics

- Carrier diffusion time ($\sim \text{ns}/\mu\text{m}$) is typically much longer than carrier transit time ($\sim 10\text{ps}/\mu\text{m}$)
- Electron and hole velocities saturated in depletion region due to high field strength
- Once away from depletion region carrier velocities fall below saturation
- Space charge barrier prevents carriers from entering the depletion region, therefore the multiple carrier effect seen in photoconductors does not occur when carrier velocities are mismatched



pn-Junction Carrier Dynamics

- Photons absorbed within one diffusion length outside the depletion region will be absorbed and the current contributing carriers will suffer both diffusion time and transit time delays
- Effect is geometry and material dependent

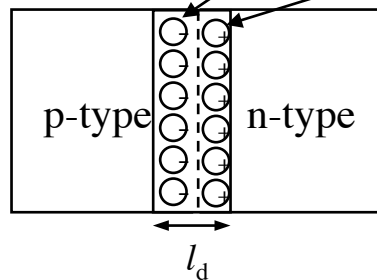


$$i_{photo}(\omega) = \frac{\eta_{DC} - \eta_{AC}}{h\nu} \frac{P_{rcvd}(\omega)}{\sqrt{1 + \left(\frac{\omega}{\omega_D}\right)^2}} + \frac{q\eta_{AC}}{h\nu} P_{rcvd}(\omega)$$

pn-Junction Carrier Dynamics

- The separation of charge in the depletion region (due to uncompensated Donors and Acceptors) leads to a capacitive effect that also impacts the detector bandwidth

Uncompensated Acceptors and Donors



$$C_j = \frac{\epsilon_0 \epsilon_r A}{l_d}$$

$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$ = vacuum permittivity

ϵ_r = semiconductor relative permittivity

A = area of depletion region

l_d = depletion region length

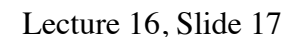
- The frequency at which the detector bandwidth rolls off by 3-dB due to the junction capacitance is

$$\omega_{RC} = \frac{1}{R_s C_j}$$

C_j = area of depletion region

l_d = depletion region length

- ECE162C, Spring 2010, Professor Blumenthal



p-i-n Photodiodes

- As with the pn-junction, the quantum efficiency is defined by the following equation, however the distance can now be integrated over the larger intrinsic region

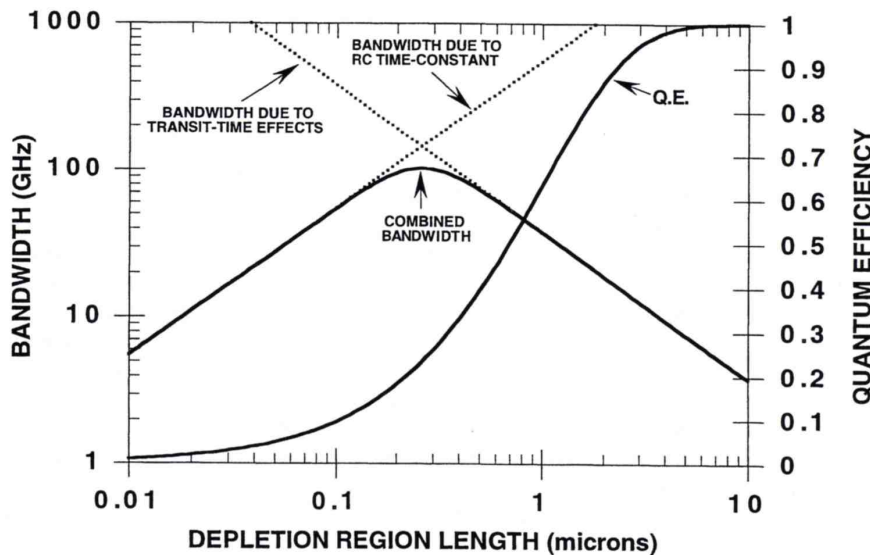
$$\begin{aligned}P_{abs}(x) &= P_i(1 - R)(1 - e^{-\alpha(\lambda)x}) \\ &= \eta(\lambda, x)P_i\end{aligned}$$

- As the depletion region length is increased, η increases, the junction capacitance C_j decreases, and the transit time τ_{trans} increases. The detector design must be optimized to maximize both efficiency and bandwidth. An estimate of the bandwidth is given by

$$B_{pin} = \frac{1}{\sqrt{\left(\frac{1}{f_{RC}}\right)^2 + \left(\frac{1}{f_{trans}}\right)^2}} = \frac{1}{\sqrt{\left(2\pi R_s \epsilon_0 \epsilon_r \frac{A}{l_d}\right)^2 + \left(\frac{1}{0.44 v_s}\right)^2}}$$

Bandwidth-Efficiency Tradeoffs in p-i-n Photodiodes

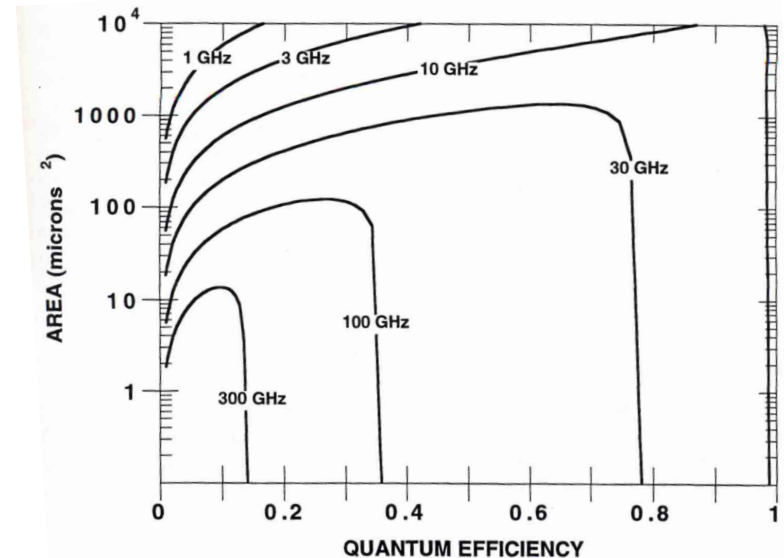
- The quantum efficiency, η , can be approximated assuming $R=0$ (high quality anti-reflection coating) and intrinsic region length l_d . $\eta = 1 - e^{-\alpha l_d}$
- For small l , bandwidth is transit time limited
- For large l , bandwidth is RC limited
- Optimal bandwidth length where two effects are equal
- QE keeps increasing with increased length



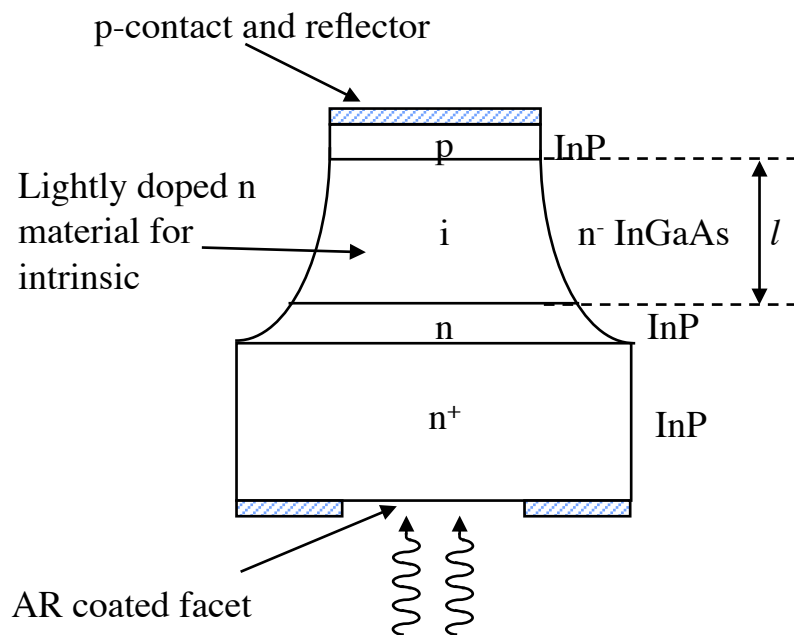
- If the detector area A and length l_d are both optimized, then bandwidth and quantum efficiency can both be maximized

$$A = \frac{l_d}{2\pi R_l \epsilon_0 \epsilon_r} \sqrt{\frac{1}{B^2} - \left(\frac{l_d}{0.44 v_s}\right)^2}$$

$$l_d = -\frac{1}{\alpha} \ln(1 - \eta)$$



Vertically Illuminated p-i-n Photodiodes



- For a double pass vertically illuminated pin detector (see left figure), the quantum efficiency is

$$\eta = (1 + re^{-\alpha l})(1 - e^{-\alpha l})$$

- When the carrier transit distance is approximately equal to l , and $\alpha l \ll 1$, the bandwidth-efficiency for a double-pass vertically illuminated pin photodiode is approximately